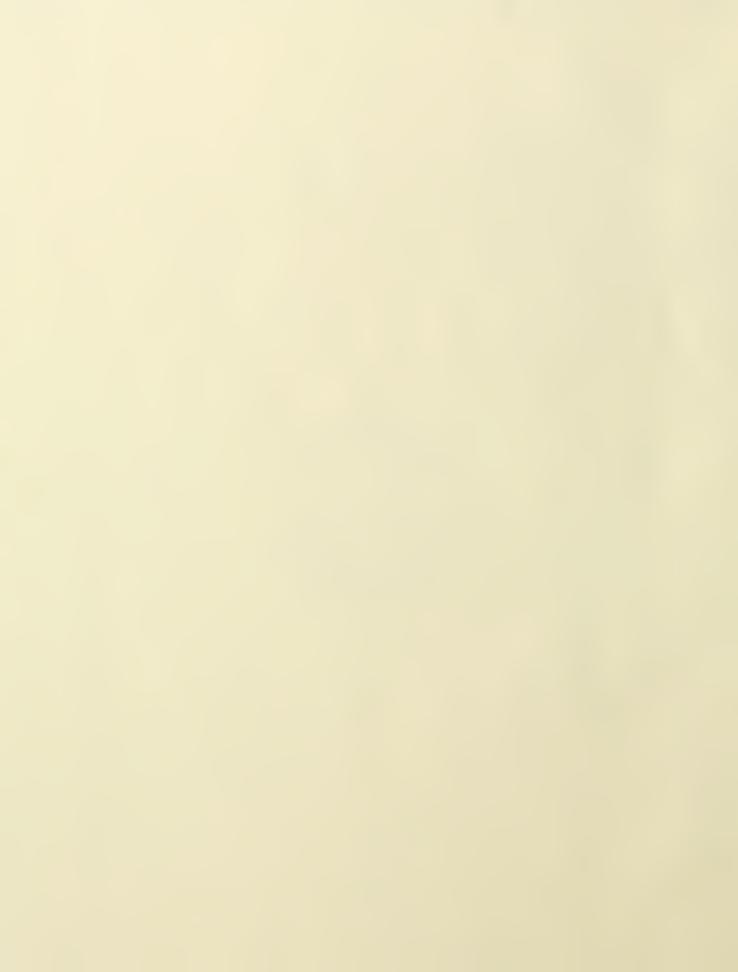working paper
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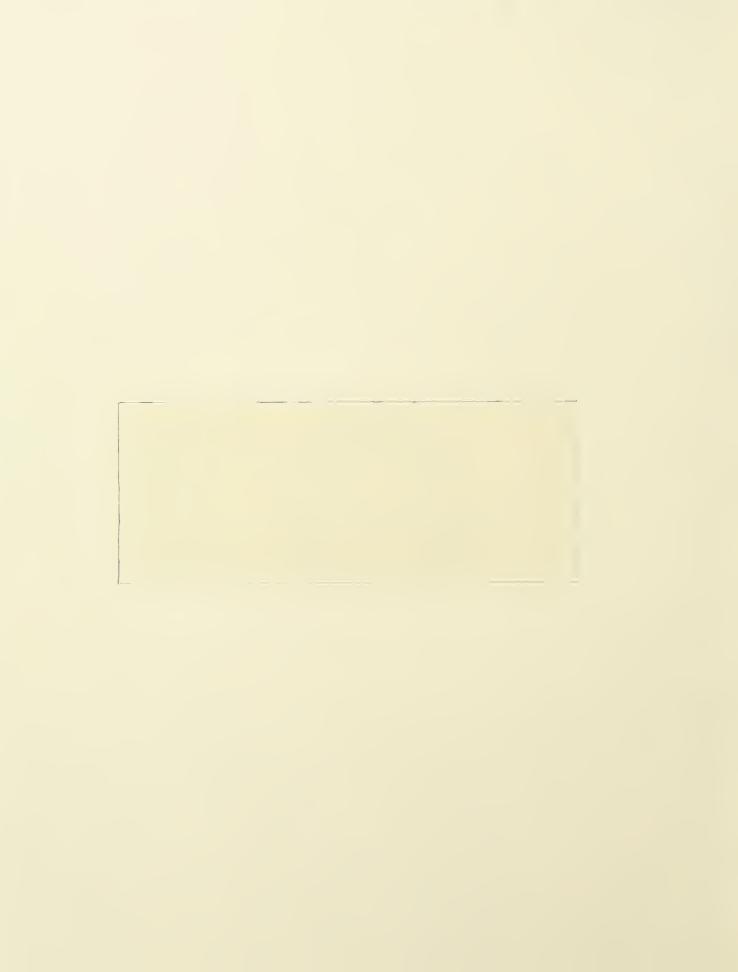
Drew Fudenberg
David K. Levine

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massachusetts
institute of
technology

50 memorial drive
cambridge, mass. 02139
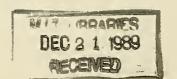
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MONOPOLY AND CREDIBILITY IN ASSET MARKETS:

AN EXAMPLE[*]

by

Drew Fudenberg

and

David K. Levine[**]

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Abstract: In a simple economy firms arise because there are economies of
scale in intertemporal production.   Consequently, agents must borrow in order
to produce.   Because overlapping generations of workers do not live long
enough to make credible promises to repay, infinitely lived patient
capitalists are the only producers.   However, the behavior of capitalists is
imperfectly monitored by the workers.   All agents are risk neutral and the
output of different capitalists is statistically independent, so adding
capitalists neither improves the monitoring technology nor enlarges the set of
feasible utilities.   Nevertheless, adding more capitalists can increase
welfare.

[**]Departments of Economics M.I.T. and UCLA.

1. Introduction

The credibility of borrowers plays a central role in capital markets. Many institutions, such as secured debt and civil law, serve to insure that borrowers repay debt. But even in the case of individual debt, and certainly in the case of sovereign debt, legal restrictions alone are not enough to insure repayment. This is especially true when debt is in the form of securities. The owners of stocks may be viewed, in the absence of voting to liquidate the firm, as lending the capital embodied in the firm to the managers. What assurance do they have that the managers will not loot the firm, converting the capital to their own use?

In this paper we consider a simple illustrative example in which firms arise because there are economies of scale in intertemporal production. This means that even though all agents have equal access to the production technology, they must borrow in order to be able to operate it. Overlapping generations of workers do not live long enough to make credible promises to repay. Consequently infinitely lived patient capitalists are the only agents who produce in the economy.

Our interest is the extent to which the first best is attainable, at least approximately, and the role played by competition between capitalists. We assume labor is inelastically supplied and that all agents are risk neutral. However, the behavior of capitalists is imperfectly monitored by the workers. Each period the capitalist consumes some of the output and distributes the remainder to the workers as dividends. The workers observe the dividend payment but do not observe the realized output, so that the capitalist's action is subject to moral hazard whenever the distribution of outcomes is stochastic.

If there is a single capitalist and output is deterministic, so there is no moral hazard, then a first best payoff is attainable when the capitalist's discount factor is close to 1. When output is stochastic, however, the first best cannot be approximated for any discount factor, and the greater the moral hazard, the greater the loss. Our main conclusion is that the loss vanishes if there are enough capitalists. In particular, competition between capitalists can make them better off.

This result is despite the fact that neither the monitoring technology nor the utility possibilities are changed by adding capitalists: Risk neutrality insures that utility possibilities do not change, while the fact that the output of different capitalists is statistically independent means that there is no improvement in monitoring. Rather, adding more capitalists promotes efficiency because the workers can "punish" one capitalist by switching their lending to another capitalist, rather than withdrawing from the market. This is important when there is moral hazard, because workers cannot distinguish between an unlucky capitalist with low output, and a greedy one. Consequently, to induce the capitalist to pay dividends when output is high, the capitalist must be punished for failing to make dividend payments when output is low. This means that even an honest capitalist faces positive probability of punishment. With a single capitalist, punishment is socially inefficient, while with several capitalists the strategy of investing in another firm when dividends are not paid provides incentives without this social cost.

This argument should be contrasted with those in Ausabel and Denecker [1989] and Gul [1986], who study sales of a durable good. In their model, a monopolist may be unable to credibly keep prices high, while with two sellers

there is an equilibrium where each seller keeps price high for fear of provoking a price war. Consequently, adding a seller reduces social efficiency by pricing willing buyers out of the market, but increases the greatest possible profit that either or both sellers can get. In our model of a lending market, adding a capitalist increases social efficiency by increasing the probability of full employment, but has no effect on the maximum profit that can be attained by a single capitalist. (It does raise the maximum combined total.)

We should also point out that there are many possible equilibria of this economy. The division of profit between capitalist and workers is undetermined, and in particular, does not depend either on the number of capitalists or the number of workers. In addition, there are equilibria that are inefficient, including the possibility of no production whatsoever. Sometimes in the macroeconomics literature this possibility is referred to as coordination failure.

A secondary goal of this paper is to exposit recently developed results on repeated games. Fudenberg, Kreps and Maskin [1989] characterized the limit set of equilibria in a repeated game with a single long-lived player facing a sequence of short-lived opponents, and showed that the folk theorem need not obtain by characterizing the equilibrium payoffs. Fudenberg and Levine [1989b] extend the characterization to the moral hazard case with multiple long-lived players. Their method of proof extends techniques developed by Abreu, Pearce and Stacchetti [1988], Fudenberg, Levine and Maskin [1989] and Matsushima [1988]. From an economic point of view, this paper is most closely related to those of Atkeson [1987] and Marimon [1988], who study issues of wealth accumulation and borrowing in a two player game with moral hazard.

In section 2 we lay out the model. In sections 3 and 4 we study equilibrium payoffs in the one capitalist and multi-capitalist cases. Section 5 considers the implications of requiring strongly symmetric punishments. These early sections of the paper focus on welfare purely from the point of view of capitalists. Section 6 addresses the issue of how well the workers do. Sections 7 and 8 consider extensions of the basic model. Both the possibility that workers live for more than two periods and the implications of allowing lending markets are considered. Section 9 concludes with a discussion of the possible role of reputation of the sort modelled in Kreps and Wilson [1982], Milgrom and Roberts [1982] and Fudenberg and Levine [1989a].

2. The Model

Trading takes place over a sequence of periods indexed $t = 0, 1, \ldots$. There are m identical capitalists. Each capitalist lives forever and has no endowment. Workers live in overlapping generations and there are qn identical workers per generation. Each worker lives two periods and is endowed with one indivisible unit of labor when young. He has no endowment when old, and consumes only then. There are no initial old workers.

Labor and the single consumption good are perfect substitutes in consumption for workers, while capitalists get no utility from labor. All agents are risk neutral. The capitalists discount future consumption with the discount factor δ which is assumed to be near one, as they are very patient. Strictly speaking, we define a capitalist's utility to be the average present value of consumption: the ordinary present value multiplied by $(1-\delta)$ so that a unit consumption stream results in a unit of utility regardless of the

discount factor. Since no consumption can take place in period zero, all present values are measured beginning in period one.

There is a production technology available equally to all agents. It converts $\ell \geq q$ units of labor in period t into the consumption good at the beginning of period t+1. There are two possible levels of output, 0 and "high". The probability of 0 output is $0 \leq 1-\pi < 1$, and the probability of "high" output is π. We choose the "high" level of output to be $\ell y/\pi$, where y>0, so that there are constant returns to scale (if more than q units of labor are used), and so that the expected amount of output remains equal to y as π changes. If several different agents are producing, the outcome of their production processes are independent of each other. This assumption is important because it means that adding producers does not improve ability to monitor production. If, for example, output were to be perfectly correlated among producers, it would be possible to design incentive schemes by comparing the output of different producers, and punishing those that claimed to have less output. On the other hand, independence means that while mean output is not changed by spreading inputs among more producers, the variance of output is reduced. Consequently, it is important that we have assumed that all agents are risk neutral: the reduced variance has no social value, and is irrelevant to the analysis.

An important aspect of this technology is that it involves an indivisibility: less than q units of labor produce no output at all. If q=1, workers can carry out production using their own labor, and this will clearly be the unique equilibrium. We wish to focus on the case where production must be centralized due to economies of scale, so we always assume q > 1. For convenience we have assumed that there are qn workers, so that

at most n different producers may operate, and all workers may contribute
equally to production.

Because production requires a one-period wait, old workers cannot
produce. It is evident that they will therefore consume any of the
consumption good they have or are given. Further, young workers have no
endowment of the consumption good. Consequently, only capitalists can hire
labor, for only they both will (potentially) have output next period, and an
incentive to use it to pay back the young workers. Indeed, since capitalists
are the only agents who can be trusted to repay, they will be the only
producers even if the workers have access to a superior technology. This
stark conclusion stems from our assumption that workers live only two periods:
the consequences of additional periods of life is discussed below.

In this production process, the expected output per unit of input is y.
We will assume that y > 1 so that the expected output is greater than the
labor input. Since all agents are risk neutral, efficiency requires that no
labor be consumed directly.

When young, each worker must decide to whom (including himself) to give
his indivisible labor input. Since workers cannot be trusted to repay loans,
we simplify the analysis by assuming the feasible actions A_2 for each worker
$j = 1,2,\ldots,qn$ are the m+1 points corresponding to either consuming the
labor input, or giving it to one of the m capitalists. The following
period, capitalist i must decide what to do with his output. (Old workers
need not decide how to dispose of their output: they never have any.) Our
basic premise is a form of limited anonymity. A capitalist can tell if
someone worked for him last period, but no further identification is possible.
Let J_i be the realized number of workers providing labor to capitalist i.

If output is zero no decision is involved. If output is "high" the capitalist consumes a fraction $(1-\phi)$ of it, with the remainder being equally divided as dividends among the workers who previously provided him with labor. Because of limited anonymity, it is not possible to pay them different amounts. Similarly, the choice of ϕ can depend on the total number of workers $J_{\hat{i}}$, but not on who they are. For simplicity, we assume that the number of workers who previously worked for other capitalists is observed after current payments are made, but before current labor is supplied. This means that $\hat{i}$'s choice of ϕ can not depend on J_i for capitalists $i \neq \hat{i}$.

For a given choice of ϕ, since high output means $J_i y/\pi$ units, the capitalist gets $(1-\phi)J_i y/\pi$ units to consume, and each worker who worked for that capitalist gets $\phi y/\pi$. For simplicity, we assume that $\phi \in \Phi$ a finite subset of the unit interval. We also assume that $0 \in \Phi$, so that the capitalist always has the option of paying no dividends.

Notice that we have assumed that the capitalist cannot pay either workers who do not work for him, or other capitalists. This rules out workers using capitalists as banks, or capitalists loaning money to each other. While the analysis of Bulow and Rogoff [1989] may seem to suggest that this is not an innocuous simplification, we show below that allowing a lending market does not affect our conclusions.

All observe to and by whom labor input is provided, and the amount of payment made if any. However, the level of output and the amount consumed is purely private information.

The Repeated Game: The consumption good is assumed to be completely perishable, and may not be used as a substitute for labor in the production

process. This simplifies the analysis somewhat: since workers wish to

consume only when old, and the consumption good is perishable, there is no

purpose in transferring the consumption good to young workers. As a result,

there is no possible trade between old and young workers, and the economy is

in effect one with non-overlapping generations of workers. We may therefore

view it as a repeated game between an infinite sequence of workers and the

capitalists, each worker playing once, and each capitalist playing repeatedly.

We discuss below the cases where lending is possible, and where workers live

more than two periods: these cases cannot be reduced to a repeated game.

The action space for a worker $j=1,\ldots,n$ is the finite set A_2

consisting of $m+1$ points: keep the labor input, or provide it to capitalist

i. For capitalist $i=1,\ldots,m$ the action space is $A_1=\Phi^{qn}$, that is, a share

of output to be paid contingent on the number of workers than provide labor.

Let $A=(A_1)^m\times(A_2)^n$ be the space of pure action profiles. For each $a \in A$ we

may then compute $g(a)$ to be the vector of expected stage game payoffs to

each agent. For example, if each worker keeps his labor, each worker gets a

payoff of 1 and each capitalist 0. Since a denotes pure action profiles,

we use α to denote mixed action profiles in which each agent plays a

probability distribution over pure actions.

Because there are indivisibilities, as pointed out by Prescott and

Townsend [1984], lotteries may be welfare improving in this economy. Indeed,

the type of labor indivisibility assumed here has been modelled using

lotteries by Rogerson [1985], and in the work on real business cycles by

Hansen [1985]. For this reason, and because it simplifies the analysis, we

assume that at the beginning of each period a public randomization device is

operated. For those who do not like lotteries, we point out that if the

discount factor of capitalists is close enough to one, our results are independent of whether or not there are lotteries: as Sorin [1986] and Fudenberg and Maskin [1988] point out, deterministic alternation between different allocations can serve as well. However, public randomization does strengthen and simplify the analysis for discount factors that are not near one.

The public history of the game at t, h_t, is a record of realized labor supply to each capitalist, payments made and the outcome of the public randomization device. Each capitalist has available also his private history h_{it} of the outcome of his own output. A strategy for capitalist i is a family of maps σ_{it} from public and private history to probability distributions over the space of actions A_1. A strategy for a generation t worker is a map σ_{jt} from public histories to probability distributions over the space of actions A_2. In a perfect public equilibrium of this game, each capitalist's strategy depends only on the public history, and each agent's strategy is a best response to his opponents following each public history. Note that it is irrelevant whether when considering best responses we restrict capitalists to strategies that depend only on the public history: given that all other players are playing that way, it is optimal for the capitalist to do so as well.

For each δ there corresponds a nonempty set of perfect public equilibria, each with a corresponding present value vector $v \in \Re^m$ for the capitalists. We let $E(\delta)$ denote the set of all such present value vectors, and refer to it as the set of equilibrium payoffs. Our interest is in relatively patient capitalists, sufficiently patient that they will invest in building a reputation for repayment. The extreme case is the limit as $\delta \rightarrow 1$.

The fact that public randomizations are possible means that $E(\delta)$ is a convex set. Both Abreu, Pearce and Stacchetti [1988] and Fudenberg, Levine and Maskin [1989] show that this implies $E(\delta)$ increases in δ. We define, therefore, the set of limit equilibrium payoffs $E = \text{closure}(\cup E(\delta))$.

Feasible Payoffs: Because workers can consume their own labor, no worker will provide labor to a capitalist who is expected to repay less than one unit per capita. For this reason, we define the (capitalists') feasible set to be the convex hull of all payoffs generated by action profiles that provide each worker who works at least one unit in expected value. Consequently, the highest feasible aggregate payoff for the capitalists is $v^* = qn(y-1)$. The (capitalists') feasible individually rational set V is the subset of the feasible set which gives each capitalist at least 0, the least he can guarantee himself against the play of all other agents. Consequently, we see that

$$V = \{ \, v \in \mathfrak{R}^m_+ \mid \Sigma^m_{i=1} \, v_i \leq qn(y-1) \, \}.$$

Naturally, $E(\delta) \subseteq V$ for all δ. Moreover, all points in the convex set V are feasible through the use of public randomizations to allocate workers to capitalists.

If E contains a point in which the combined profit of the capitalists is $v^* = qn(y-1)$, we say that the first best is attainable. (This does not imply that $E(\delta)$ contains such a point for any $\delta < 1$, merely that such a point is approached as $\delta \rightarrow 1$.) Note also that this is the first best only from the narrow point of view of the capitalists: if it is not attained, it is possible that the equilibrium is still first best from the broader point of view that includes the workers welfare. We discuss this issue below.

Focus now on the one shot game, that is, the case $\delta=0$. Regardless of the number of capitalists and workers the unique equilibrium is that the capitalists never repay and all workers consume their own labor. As a result all capitalists get 0. As is standard in repeated games, this is an equilibrium for all discount factors, although it need no longer be unique. We refer to it as the one-shot equilibrium.

3. One Capitalist

In the case where there is a single capitalist, or the case of symmetric equilibrium with many capitalists, the complete set of equilibrium payoffs $E(\delta)$ may easily be computed.

The one-shot equilibrium shows us that $0 \in E(\delta)$, and individual rationality on the part of the capitalists implies that $E(\delta) \geq 0$. Since a public randomization device is available, the set of equilibrium payoffs must be convex, and from the work of Fudenberg and Levine [1983], it must be closed. We conclude that it must be a closed interval $E(\delta) = [0,e(\delta)]$.

To characterize the set of equilibrium payoffs, we suppose that α is the strategy profile played in the first period of an equilibrium yielding the capitalist a payoff of v_i. The workers' strategies α_{-i} representing their labor supply decision in the first half of the period must be a best response to the strategy of the capitalist. The capitalist's strategies need not be a best response to α_{-i}, since the capitalist is prepared to sacrifice short run profit to increase future expected profits.

Following the play of α_{-i} the labor supply decision J_i of the workers are revealed. Output is realized, and observed by the capitalist only. He must then in the second half of the period determine how much to pay

the workers. If there is no output $\psi_i=0$ is returned to the worker. If output is "high" the capitalist pays a share $\phi_i=\psi_i$ is returned to the workers. The workers observe the realized share ψ_i, but not the "intended" ϕ_i. However, ψ_i does reveal ϕ_i unless $\psi_i=0$.

Perfect public equilibrium requires that strategies from period two on also form a perfect public equilibrium. The particular equilibrium can depend on the first-period labor supply J_i and the dividend ψ_i only. Let $w_i(J_i,\psi_i) \in E(\delta)$ denote the capitalist's payoff in this equilibrium.

Recall that v_i was the expected present value received by the capitalist in this equilibrium, and that J_i is the realized number of workers that provide labor. Supposing that J_i has positive probability, let then $v_i(J_i)$ be the expected present value conditional on the number of workers that provide labor. If the capitalist's strategy in period one given J_i, α_i places positive weight on playing ϕ, so that $\alpha_i(\phi|J_i) > 0$, it must be that

(3.1) $\quad v_i(J_i) = (1-\delta)(1-\phi)J_i y + \delta \ [(1-\pi)w_i((J_i,0) + \pi w_i(J_i,\phi)]$

$\qquad \alpha_i(J_i,\phi) > 0.$

In addition, equilibrium requires that no capitalist be able to gain by choosing a different first period action for any J_i that has positive probability. Consequently

(3.2) $v_i(J_i) \geq (1-\delta)(1-\phi)J_i y + \delta [(1-\pi)w_i((J_i,0) + \pi w_i(J_i,\phi)]$

for all $\phi \in \Phi$.

Following Radner, Myerson and Maskin [1986], it is apparent that the best equilibrium payoff for the capitalist, $e(\delta)$, can have continuation payoffs no more than $e(\delta)$: The continuation payoffs must themselves be an equilibrium. It follows that $e(\delta)$ is at most the solution of the linear programming problem

(LP) $\max_{\alpha_i(J_i), w_i(J_i,\phi)} \sum_{\phi \in \Phi} \alpha_i(\phi | J_i) v_i(J_i)$

subject to (3.1), (3.2) and

(3.3) $0 \leq w_i(J_i,\phi) \leq v_i(J_i)$.

We denote the solution of this problem $k(\alpha,\delta)$.

Conversely, suppose that from some k and all $v_i \in [0,k]$ we can find a profile α such that the workers are playing a best response to α_i and such that for every positive probability J_i there are numbers $w_i(J_i,\phi) \in [0,k]$ satisfying (3.1) and (3.2). Following Abreu, Pearce and Stacchetti [1986], we can then work the equations forward to construct strategies that are a perfect public equilibrium. Combining these observations yields

Theorem 3.1: With one capitalist $E(\delta) = [0,e(\delta)]$, where $e(\delta)$ is the supremum of $k(\alpha,\delta)$ over all α such that the workers are playing a best response to the capitalist.

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Solution of (LP): Fix a profile α, and a J_i that occurs with positive probability. Because the constraints for different values of J_i do not interact, we can proceed by maximizing $v_i(J_i)$ separately for each J_i. For the moment, then, we focus on a fixed value of J_i and omit it as an argument in α_i, w_i and v_i.

Observe that if $\alpha_i(\phi)=0$ and $\phi \neq 0$, then the punishment $w_i(\phi)$ does not occur in equilibrium and so has no cost. Consequently, for such ϕ we should choose $w_i(\phi)=0$. We then see that if the constraint corresponding to $\phi=0$ is satisfied, the constraints for $\phi \neq 0$ and ϕ not in the support of α_i will be satisfied. In other words, we may ignore those constraints.

This LP problem has a simple structure. Because the capitalist's action is revealed if output is positive, only $w_i(0)$ connects the different constraints. Ignoring $\phi=0$ for the moment, (3.1) shows that to maximize v_i for a particular ϕ, we should take $w_i(\phi)$ as large as is consistent with the constraints (3.2) and (3.3). Let $\bar{\phi}$ be the largest value of ϕ in the support of α_i. We see then that we should choose $w_i(\bar{\phi})=v_i$. Smaller values of ϕ will necessarily get smaller values of $w_i(\phi)$, since the binding constraint is (3.2) not (3.3). In particular (3.2) binding for $\phi=0$ implies that

(3.4) $\quad w_i(0) = [v_i - (1-\delta)J_i y]/\delta$.

Combining (3.4) with (3.1) at $\bar{\phi}$, and using $w_i(\bar{\phi}) = v_i$ we find

(3.5) $\quad v_i = J_i y[1 - \bar{\phi}/\pi]$.

So far we have ignored the constraint $w_i(\phi) \geq 0$. If the value $w_i(0)$ calculated from (3.4) is negative, then no solution to the LP problem exists for the given α. Consequently, we see that the solution (3.5) is valid provided

(3.6) $\delta \geq \bar{\phi}/\pi$.

We now wish to maximize $k(\alpha,\delta)$ over α that are incentive compatible for workers. The incentive constraint for the workers implies that the expected value of ϕ must be at least $1/y$ if $J_i > 0$ with positive probability. (Otherwise, $v_i = 0$.) Since (3.5) is decreasing in $\bar{\phi}$ and (3.6) increasing, the maximizing strategy for the capitalist strategy should assign probability one to

(3.7) $\hat{\phi} \equiv \min \{ \phi \in \Phi \mid \phi \geq 1/y \}$.

One consequence of this is that the capitalist need not use a mixed strategy. The only reason to mix in this game is to force the worker down to the reservation wage. Roughly speaking, mixing introduces a form of moral hazard which keeps the capitalist from benefiting from the reduction in wages. Indeed, in the case $\pi=1$, Fudenberg, Kreps and Maskin [1989] show that the best the capitalist can do is to get the worst payoff in the support of his mixed strategy. We have just showed that this result holds true even with exogenous moral hazard.

Finally, if α_i satisfies (3.7), workers are willing to work, so

$k(\alpha,\delta)$ is maximized over α_{-i} by having all workers work: that is, $J_i = qn$ with probability one. We conclude

Corollary 3.2: With one capitalist if

$$\delta < 1/\pi y + (1/\pi)(\hat{\phi}-1/y) \quad \text{then} \quad e(\delta) = 0,$$

$$\delta \geq 1/\pi y + (1/\pi)(\hat{\phi}-1/y) \quad \text{then}$$

$$(3.8) \qquad e(\delta) = qn \ [(y-1) - (1/\pi-1) - (y/\pi)(\bar{\phi}-1/y)] \equiv e^*.$$

One consequence is that if $\pi y < 1$ no equilibrium improves on the no production outcome. When production is possible, the capitalist's payoff falls short of the first best, $v^* = qn(y - 1)$, for two reasons, corresponding to the last two terms in (3.8). The first term, $qn(1/\pi-1)$ reflects the cost of providing the capitalist with incentives under moral hazard. If $\pi = 1$, this is equal to 0. The second term $qn(y/\pi)(\hat{\phi}-1/y)$ is less interesting. It reflects the fact that the capitalist cannot hold the workers to their reservation wage using a pure strategy, and as we observed above, the capitalist cannot improve his payoff by mixing. If we make the not unreasonable assumption that $1/y \in \Phi$, this term vanishes.

If $\pi = 1$ and $1/y \in \Phi$, then neither moral hazard nor discreteness poses a problem, and if $\delta \geq 1/y$ the capitalist can attain the first best. Conversely, if these conditions fail, equilibrium payoffs remain bounded away from the first best, even as $\delta \to 1$.

4. The General Case

Recall that V is the socially feasible individually rational set. Naturally, $E(\delta) \subseteq V$. Moreover, we already know that $E(0) = \{0\}$. The fact that public randomization is possible means that $E(\delta)$ is a convex set. Both Abreu, Pearce and Stacchetti [1988] and Fudenberg, Levine and Maskin [1989] show that this implies $E(\delta)$ increases in δ. We may summarize by the following

Theorem 4.1: If $\delta' \geq \delta$, then $\{0\} = E(0) \subseteq E(\delta) \subseteq E(\delta') \subseteq E \subseteq V$.

It remains to characterize E.

An important feature of this game is that the pair (ψ_i, J_i) is what Fudenberg and Levine [1989] refer to as a "personal outcome." This means that the distribution of the vector (ψ, J) conditional on (ψ_i, J_i) does not depend on capitalist i's action a_i: the personal outcome of capitalist i is a sufficient statistic for his information about the outcomes of opponents. Intuitively, this means that the problem of choosing continuation payoffs to provide incentives for different capitalists decouples into the problem of providing each individual capitalist with incentives. Moreover, we showed above in the one capitalist case that we can construct continuation payoffs $w_i(J_i, \psi_i)$ such that if δ is close enough to one and $\hat{\phi} < \pi$, he will pay the reservation wage in expected value, and all the workers will work for him. Consequently, in the terminology of Fudenberg, Levine and Maskin [1989], this profile is "enforceable." The following theorem is proven in Fudenberg and Levine [1989b]:

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Theorem 4.2: $E = \{0\} \cup \{ v \in V \mid v_i \leq k \}$ for some k.

Moreover, Fudenberg and Levine [1989b] show that k is the supremum of $k(\alpha)$ over α such that the workers are playing a best response to the capitalists. Here, $k(\alpha)$ is the solution to the same problem (LP) as in the one capitalist case, except that the constraint $w_i(\phi) \geq 0$ is not binding because we are dealing with the limit as $\delta \to 1$. The solution without this constraint is necessarily independent of the discount factor, provided only that $\delta > 0$.

Clearly the best any capitalist can do will be when the others set $\phi = 0$, in which case that capitalist will be the only one to get any labor. This takes the other capitalists out of the picture entirely, and reduces the problem of finding k to exactly the one capitalist problem. Recall that E is the limit of $E(\delta)$ as $\delta \to 1$. We conclude

Theorem 4.3: $E = \{0\} \cup \{ v \in V \mid v_i \leq e^* \}$ with e^* given in (3.8).

Even though each individual capitalist's highest equilibrium profit is no greater than if he were the only capitalist, the aggregate profit to all capitalists can be greater than if there is a single capitalist. This reflects the fact that workers can now "punish" each capitalist by working for a different one, rather than by shutting down the whole economy. In fact, if there are enough capitalists, the limit set E will contain a point where the capitalists' profit is the first-best level $v^* = qn(y-1)$. In this case, we say that the first best is attainable. From theorem 4.3, we may calculate

Corollary 4.4: The first best is attainable with one capitalist if and only if $1/y \in \Phi$ and $\pi = 1$. With $m > 1$ capitalists, the first best is also attainable if $\pi \geq \hat{\phi}$ and

$$m \geq (y-1)/y(1-\hat{\phi}/\pi)$$

which are necessary and sufficient.

This shows that the greater the number of capitalists, the broader the range of parameters π and y for which the first best is attainable.

5. The Symmetric Case

We now briefly follow Abreu, Pearce and Stacchetti [1986], and restrict attention to purely symmetric pure strategy equilibria.

In a symmetric equilibrium each capitalist gets the same number of workers and the same payoff $v_i = v$, and in each continuation equilibrium, each capitalist gets the same amount. As in the analysis of the one capitalist case, it is immediate that the set of symmetric equilibrium payoffs, $\underline{E}(\delta) = [0, \underline{e}(\delta)]$.

Using the same type of argument as in the single capitalist case, we may find $\underline{e}(\delta)$ as the maximum of $k(a,\delta)$ over a in which the workers are playing a best response to the capitalists resulting in each capitalist receiving the same labor supply J, and all capitalists are playing the same pure strategy $a_i = \phi^*$. Since the capitalists are playing pure strategies, and we need consider only deviations by a single player, we may represent the common continuation payoff by $w(\phi, M)$ where M is the number of capitalists that pay ϕ^*. Let $\pi(M)$ be the probability that exactly M of $m-1$ capitalists simultaneously have output:

$$(5.1) \quad \pi(M) = \begin{pmatrix} m-1 \\ M \end{pmatrix} (1-\pi)^{m-M-1}\pi^M.$$

Then the incentive constraints that characterize $k(a,\delta)$ are

$$(5.2) \quad v \geq (1-\delta)\phi Jy + \delta \sum_{M=0}^{m-1} \pi(M) [(1-\pi)w(0,M) + \pi w(\phi,M)]$$

with equality if $\phi = \phi^*$,

and $k(a,\delta)$ is the maximal value of v. We should clearly set $w(\phi,M)=0$ if $\phi \neq \phi^*$, since this punishment need not be carried out in equilibrium. This means that the strategy $\phi = 0$ is better than any strategy other than $\phi = \phi^*$. Consequently, we may define $w(M)$ to be the continuation present value corresponding to exactly M capitalists paying ϕ^* and the remainder paying zero, and rewrite (5.2) as

$$(5.3) \quad v = (1-\delta)\phi^*Jy + \delta \sum_{M=0}^{m-1} \pi(M) [(1-\pi)w(M) + \pi w(M+1)]$$

$$v \geq (1-\delta)Jy + \delta \sum_{M=0}^{m-1} \pi(M) w(M).$$

If we now define $w_i(0) = \sum_{M=0}^{m-1} \pi(M) w(M)$ and $w_i(\phi^*) = \sum_{M=0}^{m-1} \pi(M) w(M+1)$, then it is clear that every feasible solution of this problem gives rise to a feasible solution of problem (LP) we solved in the single capitalist case. If $\delta < 1/\pi y + (1/\pi)(\hat{\phi}-1/y)$ the only solution to that problem was 0, so this is true in the symmetric case as well. Moreover, the solution to the single capitalist LP problem never exceeded the value in (3.8), so the solution to this problem does not either. When is that value actually attainable in the symmetric case? Recalling that the solution in one capitalist case was to set

$w_i(\phi^*) = v$, we see that we must choose $w(M) = v$ for $M > 0$. Consequently the punishment occurs only when $M = 0$, that is, none of the capitalists pay. This occurs with probability $\pi(0) = (1-\pi)^m$. Moreover, both Abreu, Pearce and Stacchetti [1988] and Fudenberg, Levine and Maskin [1989] show that the fact that $\underline{E}(\delta)$ is convex implies that it increases in δ. Restricting attention to the case where the reservation wage is attainable in pure strategies, we conclude:

Theorem 5.1: Suppose $1/y \in \Phi$, and we consider only symmetric pure strategy equilibria. If $\delta < 1/\pi y$ then k=0. If $\delta \geq 1/(1-(1-\pi)^m)y$ then $k = (qn/m)[y-1/\pi]$. For $1/\pi \leq \delta \leq 1/(1-(1-\pi)^m)y$, $0 \leq k(\delta) < (qn/m)[y-1/\pi]$ is a non-decreasing function of δ.

In particular, when there is moral hazard, for a range of parameter values, the combined profit of the capitalists is less than the profit of a single capitalist. This is the opposite result from the general case, with the reservation that the symmetric analysis shows that the discount factor at which maximal joint payoffs are obtained goes up with more capitalists and the maximum does not change, while the general analysis shows that the maximum increases with the number of capitalists. However, we cannot say whether or not higher discount factors are required to approach the maximum in the general case.

The problem with the symmetric case is not that a symmetric payoffs are unreasonable given the symmetry of the situation, but rather that symmetric punishments are not a very good idea. If too many of the capitalists fail to pay, all of them are punished. Since through random mischance, it will

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periodically happen that many capitalists will simultaneously have no output,
these punishments have a cost. A better arrangement would be to punish only
those capitalists that actually failed to pay, by having the workers transfer
their labor to those that did pay. Since the punishments take the form of a
transfer payment, there is no social cost. which is why non-symmetric
punishments are more efficient.

6. Worker Welfare

Until now, our focus has been on efficiency from the narrow point of
view of the capitalists. In particular, with moral hazard and a single
capitalist, we have argued that the first best is not attainable. This simply
means that the capitalists can not get as much rent as they could from
precommitment. We now argue that the terminology is not misleading: in fact
with moral hazard and a single capitalist, from the broader welfare point of
view that includes the workers, the first best is not attainable.

The first best from the joint perspective of workers and capitalists
requires full employment: that is, that all qn units of labor available to
the economy be employed each period. Since the workers in each period are
different, the way in which this is divided between worker and capitalist or
different workers within each period is welfare neutral. We now argue that in
any equilibrium unemployment must occur infinitely often, contradicting full
efficiency.

Suppose in fact that full employment always occurs in equilibrium. If
v_i is an equilibrium present value when there is full employment, it must not
be optimal to pay zero. If $w_i(0)$ is the continuation present value when no
payment is made, this means that

(6.1) $v_i \geq (1-\delta)qny + \delta w_i(0)$.

This may be rewritten as

(6.2) $v_i \geq (1-\delta)qny + \delta(v_i - [v_i-w_i(0)])$.

Rearranging terms then yields the inequality

(6.3) $v_i - w_i(0) \geq ((1-\delta)/\delta) [qny - v_i]$.

However we know that because workers are individually rational, in any equilibrium, $v_i \leq qn(y-1)$. It follows from (6.3) that

(6.4) $v_i - w_i(0) \geq ((1-\delta)/\delta) qn$.

If $\pi < 1$, it then follows that every period in which there is full employment, there is probability $(1-\pi)$ that the continuation present value drops by at least $((1-\delta)/\delta)qn$. By assumption, the continuation equilibrium with present value $w_i(0)$ also has full employment. This can occur only a fixed number of times T (the number depending on the discount factor) before the constraint $w_i(0) \geq 0$ is violated. Since the probability that this happens is $(1-\pi)^T$, there is a positive probability that there is less than full employment. We conclude that with moral hazard, unemployment must occur infinitely often.

In this calculation, it is clear that as $\delta \to 1$ it is possible to

construct equilibria in which there is full employment for increasingly long periods of time. It is tempting to argue that this means "approximate efficiency" is attained as we approach the limit. However, as the period of full employment grows longer, the capitalist's discount factor grows closer to one at exactly the same rate, so if we use it in "discounting future unemployment" we see that the present value of future unemployment remains unchanged.

We should note that it is not true that in all equilibria workers get their reservation wage. If Φ contains only two points, $0, \phi \in \Phi$, and $\phi > 1/y$, we know from the solution of the LP problem in the one capitalist case that if δ is close enough to one, there exists an equilibrium in which ϕ is played in the initial period. Moreover, as long as $\phi \in \Phi$, regardless of how many points Φ has, this same equilibrium can be enforced for δ near enough one by punishing the capitalist with zero if he ever plays anything other than ϕ or 0.

7. <u>Lending Between Capitalists</u>

Bulow and Rogoff [1989] argue that in the case of sovereign debt, the ability of a borrower to make deposits in a bank at the competitive interest rate means that reputation alone is not enough to create a market for debt: there is always a point at which it would be optimal for the borrower to renege on the debt and deposit the proceeds in a bank. This suggests that if we expand the model by allowing permitting all capitalists and workers to freely transfer consumption to one another, and there are two or more monopolists, the equilibrium set will collapse to the static equilibrium. This is not the case: Any equilibrium of the model without transfers becomes

an equilibrium of the model with transfers, provided all capitalists follow the strategy of never repaying any loans. This contrasts with the Bulow-Rogoff assumption that banks must repay all depositors. However, unless there are effective legal restrictions, it is not obvious why banks that accept deposits from defaulters should repay them as Bulow and Rogoff require. If the deposits are secret, then no one will ever know that the bank refused to repay. If they are public, then the bank may only lose the premium it is charging defaulters over other borrowers (nothing in the Bulow-Rogoff analysis).

If all capitalists are forced to pay a competitive rate of return on loans of $(1-\delta)/\delta$ per period, but are allowed to default on payments to workers, then intuitively the Bulow-Rogoff analysis does hold, and equilibrium set will collapse: Each capitalist will renege on his initial payment to workers. He will then use the proceeds to pay young workers in advance for their labor. The workers who have no use for this consumption will deposit it with capitalists, who will necessarily repay the loan. Of course this means that the initial workers will not agree to work, knowing that they will not be paid, and so no production will ever take place.

There are several complications with formalizing this analysis. Since capitalists must choose between giving consumption to young or old workers, the generations now overlap in a non-trivial way, and we are no longer dealing with a repeated game. A more substantial problem is that capitalists cannot be forced to pay $(1-\delta)/\delta$ on loans: there is no storage technology, and even if we make consumption a perfect substitute for labor in production, there may not be any output with which to repay. However, these objections are minor relative to the most significant point: Why does it make sense to assume that

capitalists are obligated to repay loans, but can renege on payments to workers? The mechanism by which relatively efficient outcomes are obtained is precisely by having capitalists renege on loans, but not on payments to workers.

Since many features of our model are unrealistic, we do not contend that our results invalidate the analysis of Bulow-Rogoff. We feel that the fact that lending is enforced by the threat of exclusion from future borrowing in our completely specified general equilibrium environment leads to reservations about the generality of their analysis.

8. Longer Worker Lifetimes

We now address the robustness of our conclusions to allowing workers to have longer lifetimes. A key feature of our model was that capitalists were necessary in order to arrange production. Once workers live three periods, this is no longer true. Suppose that workers have labor endowment only when young and wish to consume only when old, but that they are also middle aged in between. This has the implication that they can engage in production when middle aged. Consider the following strategies: Each period, a middle-aged worker is picked at random from the pool of "eligible" middle-aged workers, and all young workers give him their labor. The lucky designee then produces and consumes all the output himself. Young workers agree to participate in this scheme because if they do not they are not "eligible" when they become middle aged. Under our extreme assumption of risk neutrality, this scheme actually implements the first best.

Once risk aversion is introduced a lottery scheme of this type becomes less desireable. If the indivisibilities are large, the inefficiency is large

as well: imagine compensating employees at General Motors by a small chance at all of General Motors profits for a year. Consequently, while capitalists are not necessary for production in this more general setup, they will have a role to play if indivisibilities and risk aversion are large compared to the losses caused by the capitalists' moral hazard.

9. The Role of Reputation

The analysis so far has been in terms of a repeated game. The equilibrium involves "reputation" in the loose sense that capitalists repay loans so that they will be allowed to borrow in the future. However, reputation here is purely an equilibrium phenomenon, in the sense that repayment of a loan need not necessarily signal a future willingness to repay. Kreps and Wilson [1982] and Milgrom and Roberts [1982] have introduced a more explicit sort of reputation. If the workers a priori suspect that there is a possibility that the capitalists will choose to repay regardless of future consequences, then capitalists can maintain this belief by repaying. Fudenberg and Levine [1989c] have shown that typically with a single patient capitalist, this type of explicit reputation leads him to do as well as he could through precommitment.

Unfortunately, there is an important gap in the literature, and in this model we can say relatively little. The problem, pointed out in Fudenberg and Levine [1989a] lies in the extensive form nature of the game. Suppose for simplicity that $1/y \in \Phi$, and that the single capitalist is one of two types: a rational type as described above, and a "commitment" type who always pays $\phi = 1/y$. If there is moral hazard, the rational capitalist would like to convince the workers that he will always repay, for if they believe him they

will always work, and he will get v^*=qn [y-1] rather than at most qn [y-1/π]. The problem is that there may be no opportunity to create a reputation: if the workers believe they will not be paid, they will not work, and so the capitalist will have no opportunity to prove that he will repay.

We can see this explicitly in the case without moral hazard, where π = 1. In this case it is obviously impossible to get more than v^*. Consequently, we can use a construction from Fudenberg and Levine [1989] to find a sequential equilibrium in which the capitalist gets nothing. In the initial period, the capitalist does not pay. The workers do not work unless the initial worker did work, and the capitalist did not pay him. In this case we switch to the equilibrium in which the capitalist gets exactly v^*. This rationalizes the capitalist not paying in the initial period: if he does pay, he loses utility now, and does no better in the future. On the other hand, the equilibrium in which the capitalist gets v^* calls for him to pay all the time, and workers to stop working for him if he does not. This is obviously still an equilibrium in the perturbed game, so we conclude that the introduction of explicit reputation may have no significant effect.

The above argument relied on the fact that without moral hazard the best payoff with reputation effects was already possible when explicit reputation was absent. This need not be the case with moral hazard and so the effect of reputation is still unknown. To get a feeling for the possibilities, first consider an alternative model in which the capitalist must precommit to ϕ before knowing whether or not any workers will provide labor, and in which the workers find out what ϕ was even though they did not provide any labor. Although it is hard to motivate this model, it might be imagined that the capitalist operates a second plant with a captive work force, but is forced to

pay both groups of workers the same amount. In this case the game is a simultaneous move game: the capitalist pick ϕ without knowing the whether the workers will work, and the workers choose to work without knowing ϕ. There is moral hazard, but the information about the capitalist revealed at the end of the period is independent of what the workers do. In this case the result of Fudenberg and Levine [1989c] shows that as $\delta \to 1$ the payoff going to the capitalist in all Nash equilibria approaches nq [y-1]. In particular if $\pi < 1$, then reputation will allow a sufficiently patient capitalist to do strictly better than without the possibility of explicit reputation. As a result, our construction of an equilibrium in which the capitalist gets zero fails, and there is relatively little we can say about explicit reputation in the moral hazard case.

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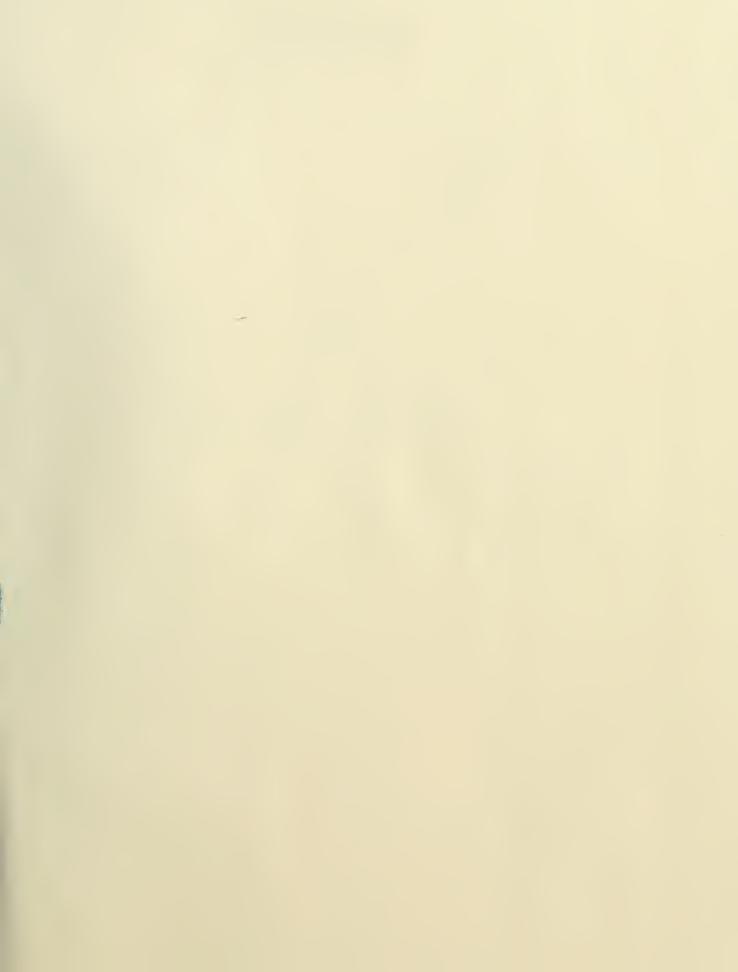Discounting and Incomplete Information," <u>Econometrica</u> 54, 533-54

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